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ELECTRONICS ENGINEERING-ECE

STUDY MATERIAL

NETWORK THEORY

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ENGINEERS INSTITUTE OF INDIA

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CONTENT

CHAPTER-1

BASIC CIRCUIT ELEMENTS & THEORY

INTRODUCTION TO CIRCUIT ELEMENT:

Electric circuit is an inter connection of electric elements.

- Charge is electrical property of atomic particles.

  Unit → Coulombs
- Charge on electron = -1.6×10^{-19}C
- Current is time rate of change of charge.

$$I = \frac{dq}{dt} \quad \text{or} \quad q = \int_0^t I\,dt$$

Unit → $\frac{C}{sec}$ or Amp

- Voltage is the energy required to move a charge from one point to another point.

$$V = \frac{dW}{dq}$$

Unit → $\frac{J}{C}$ or Volt

- Power is the time rate of change of energy.

$$P = \frac{dW}{dt} \quad \text{or} \quad W = \int P dt$$

Unit → $\frac{Joule}{sec}$

Classification of circuit element:-

(i) Unilateral and Bilateral element:-

If the element property and characteristic does not change with direction of current, then the element is called bilateral element; otherwise unilateral element.

(ii) Linear and non linear element:-

If the element satisfy homogeneity and additivity property then element is called linear element, otherwise non linear.

(iii) Active and passive Elements:

Active Elements: When the element is capable of delivering the energy, it is called active element.

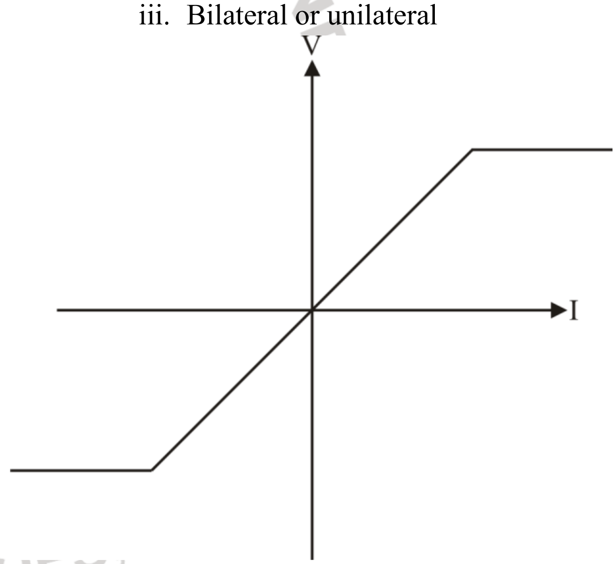
Example: Voltage source, Current source, Transistor, Diode, Op-amp etc

Passive Elements: When the element is not capable of delivering the energy, it is called passive element.

Example: Resistance, capacitor, inductor etc.

Example: Identify weather the element is:

i. Linear or non linear
ii. Active or passive
iii. Bilateral or unilateral



Solution:-

i. Nonlinear, as slope is not constant.
ii. Passive, as V/I is +ve in both quadrants.
iii. Bilateral, as characteristic is identical in opposite quadrant.

1. Resistance:

Ohm's law:- Voltage V across a resistor is directly proportional to the current i flowing through the resistor

⇒ V α i

V = iR, This constant of proportionality is called 'resistance'.

⇒ Case 1:- When R = 0 → short circuit

Then V = 0 and I = ∞
Case 2:- When R = ∞ → Open circuit
Then V = ∞ and I = 0

Key Points:

- Power in resistor is given by

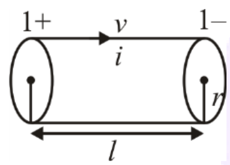
$$P = vi = i^2R = \frac{v^2}{R}$$

- Energy is then determined as the integral of instantaneous power as :

$$E = \int_{t_1}^{t_2} P dt = R\int_{t_1}^{t_2} i^2 dt = \frac{1}{R}\int_{t_1}^{t_2} v^2 dt$$

- Resistor consumes energy and converts electrical energy into heat energy.
- Resistance depends on the geometry of material and also on nature of material as:

$$R = \rho\frac{l}{A}$$



Where ρ = Resistivity ($\Omega.m$)
ρ = 1/σ (σ = conductivity)
Unit of conductivity: mho/m or Siemens/cm

- Resistivity of wire is materialistic property i.e. It does not vary with circuit geometry.
- Extension of wire result in increase in length & decrease in cross-sectional area therefore resistance of wire increases.
- When circuit is short circuit means, $R = 0$. When circuit is open, $R = \infty$.

Example: A conductor has a resistance of 3Ω. What is resistance of the same material? Which has one half the diameter and 4 times the length of the given conductor.

EXP: $R_1 = \rho\frac{l_1}{A_1}$, $R_2 = \rho\frac{l_2}{A_2}$

$$\Rightarrow \frac{R_2}{R_1} = \frac{l_2}{l_1}\frac{A_1}{A_2}$$

Now $A_1 = \frac{\pi D_1^2}{4}$, $A_2 = \frac{\pi D_2^2}{4}$

$$\Rightarrow \frac{A_1}{A_2} = \frac{D_1^2}{D_2^2} = \frac{D_1^2}{\left(\frac{D_1}{2}\right)^2} = 4 \text{ and}$$

$$l_2 = 4l_1 \Rightarrow \frac{l_2}{l_1} = 4$$

$$\Rightarrow \frac{R_2}{R_1} = 4 \times 4 = 16$$

⇒ $R_2 = 16 \times 3 = 48\Omega$

2. CAPACITANCE:

Capacitance is the property of capacitor which opposes the sudden change in voltage.

$$Q \propto V$$

$$Q = CV$$

$$\frac{dQ}{dt} = C\frac{dv}{dt}$$

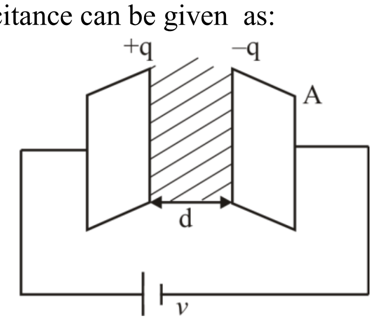
$$I = C\frac{dv}{dt}$$

$$\text{Or, } V = \frac{1}{C}\int_{-\infty}^{t} I\, dt$$

The circuit element that stores energy in an electric field is called capacitor.

Key Points:

(a) **<u>Capacitors retain the charge & thus electric field</u>** after removal of the source applied. (While inductors do not retain energy). For parallel plate capacitor, the capacitance can be given as:



$$C = \frac{\epsilon_o \epsilon_r A}{d}$$

Where A = cross-sectional area of plate

$\in_r$ = Relative permittivity of dielectric

$\in_o$ = Permittivity of free space

d = distance between plates

$$\boxed{C = \frac{8.854 \in_r A}{d} pF}$$

(b) The charge q on capacitor results in an electric field in the dielectric which is the mechanism of energy storage.

(c) Power and energy relation for capacitance are as:

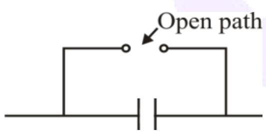
$$P = vi = vc\frac{dv}{dt} = \frac{d}{dt}\left[\frac{1}{2}cv^2\right] \qquad \left\{i = \frac{cdv}{dt}\right\}$$

$$\boxed{P = \frac{d}{dt}\left[\frac{1}{2}cv^2\right]}$$

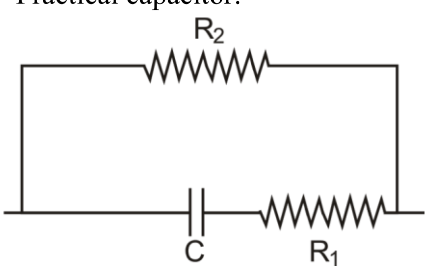
Energy $w_c = \int P\,dt = \int v\,c\frac{dv}{dt}dt$

(d) The energy stored in the electric field of capacitance is $\boxed{w_c = \frac{1}{2}cv^2}$

(e) Ideal capacitor:



Practical capacitor:



3. INDUCTANCE:

Inductor:- Inductance is the property of the inductor which opposes the sudden change in current.

Concept:- When a time varying current is flowing through the coil, then magnetic flux is induced and it is given by

$\psi \propto I$

$\psi = Li$

$N\phi = LI$

$$\boxed{L = \frac{N\phi}{I}}$$

Key Points:

(a) The flux linkage across inductor is $N\phi$. Thus $\boxed{N\phi = LI}$

(b) **Proof of equation A:** According to faraday's law, the emf induced across a inductor is directly proportional to the rate of change of flux through it.

$e = -N\frac{d\phi}{dt}$ {N = no of turns in the coil}

$$e = -N\frac{d}{dt}\left\{\frac{LI}{N}\right\}$$

$$\boxed{e = -L\frac{dI}{dt}}$$

-ve sign indicates the opposition caused by induced emf to change of flux (Lenz's Law)

(c) The power across the inductor is:

$$\boxed{P = vi = L\frac{di}{dt}i = \frac{d}{dt}\left[\frac{1}{2}Li^2\right]}$$

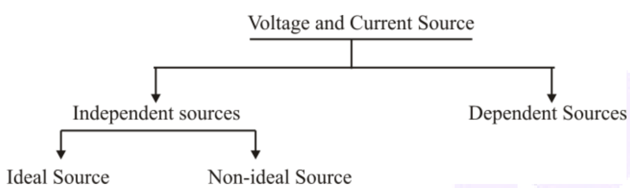
(d) Energy: $w = \int P\,dt = \int Li\,dt$

$$\boxed{w = \frac{1}{2}L\left[i_2^2 - i_1^2\right]}$$

Energy stored in magnetic field by inductor is $\boxed{w = \frac{1}{2}Li^2}$

Relationship of parameters:

Element	Units	Voltage	Current	Power
Resistance	Ohms (Ω)	$v = Ri$ (ohms law)	$i = \frac{v}{R}$	$P = vi = i^2R$
inductance	Henry (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int vdt$	$P = vi = i\frac{Ldi}{dt}$
Capacitance	Farad (F)	$v = \frac{1}{c}\int idt$	$i = c\frac{dv}{dt}$	$P = vi = vc\frac{dv}{dt}$



Voltage & Current Source:

The sources are of two types, one is independent sources and other is dependent sources:

Independent sources:

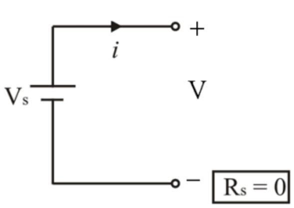
The voltage or current source in which the value of voltage or current remains constant, and does not vary with other circuit element.

Ideal & practical voltage sources:

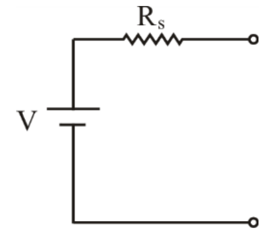
If the voltage source delivers energy at particular voltage, which is independent of source current then voltage source is ideal, otherwise practical.

Ideal:- $V = V_S$

Practical:- $V = V_S - IR_S$



Ideal Voltage Source



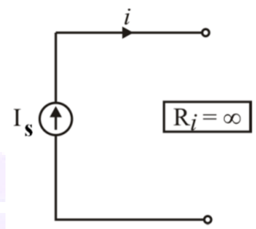
Practical Voltage source

Ideal & practical voltage current sources:

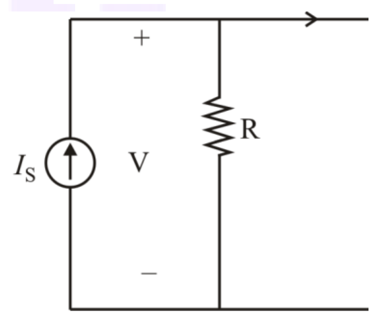
If the current source delivers energy at particular value, which is independent of source voltage, then current source is ideal, otherwise practical.

Ideal: $I = I_S$

Practical: $I = I_S - V/R_S$



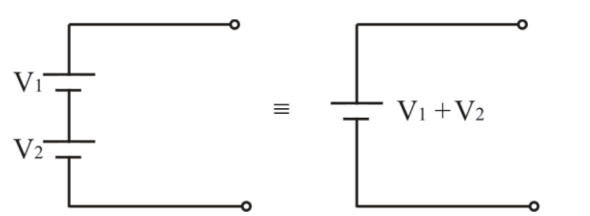
Ideal current source

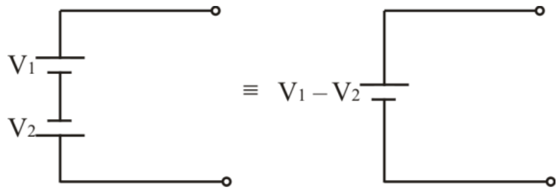


Practical Current source

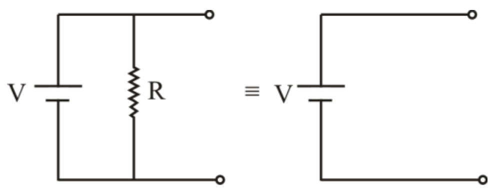
(a) In non ideal voltage source, the internal resistance of voltage source is of finite value and is always in series with voltage source.

(b) In non ideal current source, the internal resistance of current source is of finite value & is always in parallel with current source.

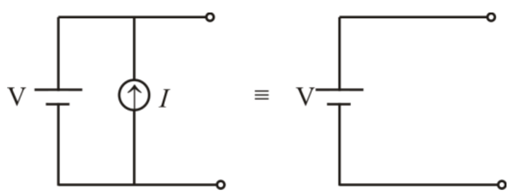




(c)



(d)



Key Points:

→ Resistance in parallel with a voltage source is redundant as the terminal voltage remains same.

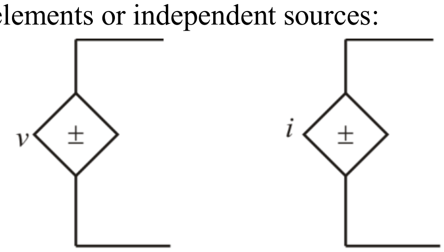
→ Resistance in series with the current source is redundant as the short circuit current in loop is independent of value of R.

→ When current sources are connected in series they should all have same value.

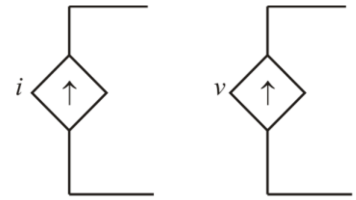
→ When voltage sources are connected in parallel they should have same value.

Dependent Voltage and Current Sources:

These are voltage and current sources whose value do not remain constant, rather varies with circuit elements or independent sources:



Voltage dependent voltage source   Current dependent voltage source



Current dependent current source   Voltage dependent current source

Distributed and Lumped Network:

In Lumped network, we can separate resistance, inductance, and capacitance separately or single element in one location is used to represent a distributed resistance.

Example: A coil having large number of turns of insulated wire has resistance throughout the length of wire but only resistance at single plane represents the distributed resistance.

In Distributed network, the circuit elements are not at one location rather they are distributed.

Example: Transmission line, the resistance, inductance and capacitance are distributed throughout the length of Transmission line.

Note: In distributed network, the circuit elements are represented as per unit length.

Non Linearity of circuit elements:

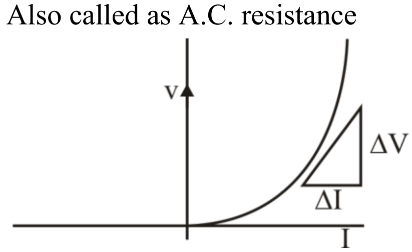
1. **Resistance Non Linearity:** If the current voltage relationship in an element is not linear, then the element is modeled as non linear resistor.

   Example: Diode, filament lamp

   (a) The non linear resistance can be given as:

   $$R = \frac{\Delta V}{\Delta I}$$

   Also called as A.C. resistance



Note: Ohm's law is valid for linear circuit elements. Also it is not valid for open circuit element because for open circuit:

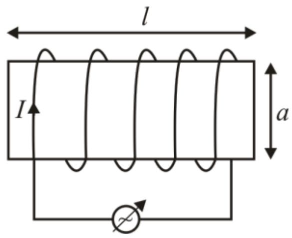
$I = 0, R = \infty$

So $V = \infty$

$$\boxed{V \neq IR}$$

2. **Inductors non linearity:** When the inductance of inductor depends on the current magnitude, then the inductor is called non linear inductor:
Example: Iron core inductor.

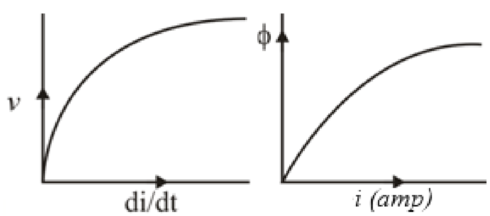
Key points:



(a) $$\boxed{N\phi = LI \Rightarrow L = \frac{N\phi}{I} = Variable}$$

(b) Also we know;

$$\boxed{V = L\frac{di}{dt} \Rightarrow L = \frac{V}{di/dt} = Variable}$$



(1) (2)

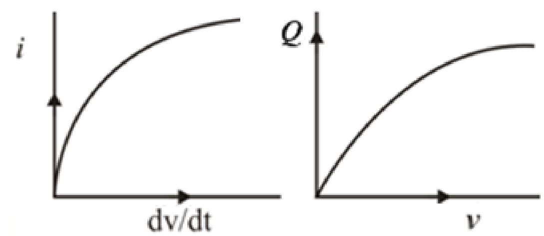
As the slope of the curve in both cases is **L** (inductance) and **L** is variable. So, the curve is not linear.

The second curve shows that after certain value of current, the flux does not increase due to saturation of iron core.

3. **Nonlinearity in capacitance:** When the capacitance of capacitor depends on voltage magnitude, then capacitor is called non linear capacitor.

$$\boxed{Q = CV \Rightarrow C = \frac{Q}{V} = Variable}$$

$$\boxed{i = C\frac{dv}{dt} \Rightarrow C = \frac{i}{dv/dt} = Variable}$$



(1) (2)

As the slope of the curve in both cases is **C** and **C** is variable. So, the curve is not linear.

Key Points:

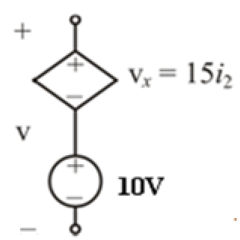
(*a*) Resistances may exhibit non linearity and not obey ohm's law due to thermal effects.

(*b*) Inductances without air core have saturation characteristics hence they lose their linearity.

Example: Obtain the voltage V in the branch shown in figure for

(a) $i_2 = 1A$ (b) $i_2 = -2A$ (c) $i_2 = 0A$.

Solution: $v = 10 + v_x$ for



(a) $i_2 = 1A$

$V = 10 + 15 = 25V$

(b) $V = 10 - 15 \times 2 = -20V$.

(c) $V = 10 + 15 \times 0 = 10V$.

CHAPTER-2

METHOD OF ANALYSIS

1. SOME BASIC TERMS:

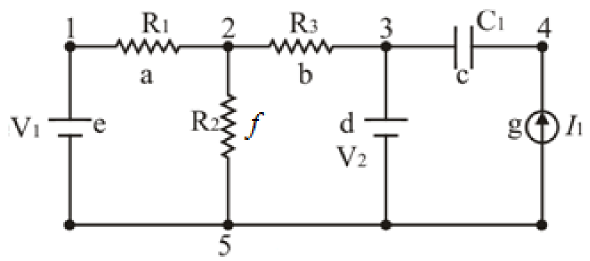
Node: Any point in a circuit where the terminals of two or more elements are connected together.

Branch: A branch is a part of circuit which extends from one node to other. A branch Represent a single element. It has two terminals.

Mesh: Any closed path which contains no other path within, called mesh.

Loop: Loop is any closed Path in the circuit. Thus a loop contains meshes but a mesh does not contain loop.

Example: Consider the following circuit:



(a) Point 1, 2, 3, 4, and 5 are nodes.
(b) a, b, c, d, e, f and g are branches.
(c) Meshes are: 1 2 5, 2 3 5, 3 4 5
(d) Loop are: 1 2 3 5 1, 2 3 4 5 2, 123451

2. KIRCHHOFF'S VOLTAGE LAW:

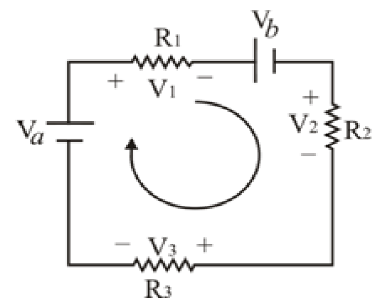
For any closed path in a network, Kirchhoff Voltage Law (KVL) state that the algebraic sum of the voltage is zero.

Key points:

(a) $\sum v(t) = 0;\ for$ Closed Path

(b) This law applies equally to *DC*, time variable sources.

Example: Write KVL equation for the circuit shown:



$$+V_a - V_1 - V_b - V_2 - V_3 = 0$$

Or $$V_a - iR_1 - V_b - iR_2 - iR_3 = 0$$

$$V_a - V_b = i(R_1 + R_2 + R_3)$$

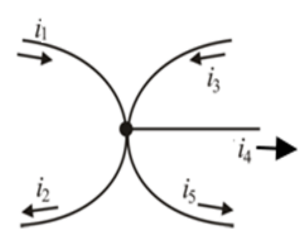
3. KIRCHHOFF'S CURRENT LAW:

KCL states that the algebraic sum of currents at any instant at a node is zero. Alternatively the sum of currents entering a node is equal to sum of currents leaving that node.

Key Points:

(a) It is based on the conservation of electric charge.
(b) $\sum i(t) = 0$
(c) Current entering node $\rightarrow$ are assigned +ve sign and current leaving node $\rightarrow$ are assigned –ve sign.

Example: Write the KCL equation for the principal node shown in fig below:



Solution: Principal node: Same as essential node.

$$i_1 - i_2 + i_3 - i_4 - i_5 = 0$$

$$i_1 + i_3 = i_2 + i_4 + i_5.$$

4. CIRCUIT ELEMENTS IN SERIES:

The 3 passive circuit elements in series connection have same current i. The voltages across elements are v_1 , v_2 , v_3.

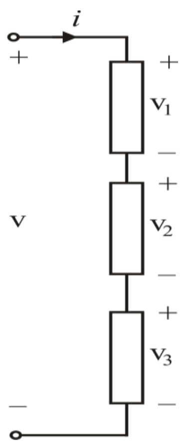
Total voltage $v = v_1 + v_2 + v_3$.

(a) **Equivalent Resistance:** When element is resistance :

$$v = i(R_1 + R_2 + R_3)$$

$$v = i\,\mathrm{R}_{eq}.$$

$$\boxed{\mathrm{R}_{eq.} = R_1 + R_2 + R_3}$$



(b) **Equivalent Inductance:** When element in above circuit is inductor then :

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt}$$

$$v = (L_1 + L_2 + L_3)\frac{di}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + L_3}$$

For any number of inductance:

$$L_{eq} = L_1 + L_2 + L_3 + L_4 + \ldots\ldots..$$

(c) **Equivalent Capacitance :** When the circuit element is capacitor in above circuit then,

$$v = \frac{1}{c_1}\int i\,dt + \frac{1}{c_2}\int i\,dt + \frac{1}{c_3}\int idt$$

$$v = \left(\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}\right)\int idt$$

$$v = \frac{1}{C_{eq}}\int i\,dt$$

Then $\boxed{\frac{1}{C_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}}$

5. CIRCUIT ELEMENTS IN PARALLEL:

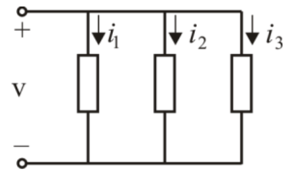
The 3 element are connected as shown in figure

(a) **Equivalent Resistance:** $i = i_1 + i_2 + i_3$

$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$$

$$i = v\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

Then $\boxed{\frac{1}{\mathrm{R}_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$



(b) **Equivalent Inductance :**

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots..}$$

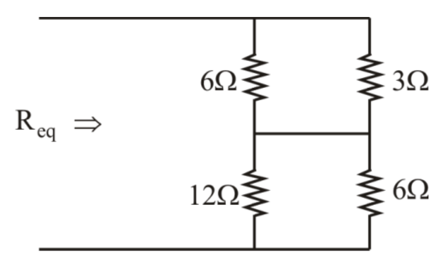
For two inductance $\boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$

(c) **Equivalent Capacitance :**

$$\boxed{C_{eq} = c_1 + c_2 + \ldots\ldots}$$

This is of the same form as resistor in series.

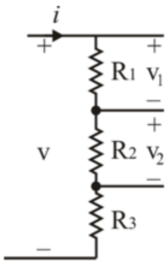
Example: Find R_{eq}



$R_{eq} = (6\|3) + (12\|6) = 6\Omega$

6. VOLTAGE DIVISION:

For resistor: A set of series-connected resistor is referred as a voltage divider.



This concept is applicable to *n* number of resistance.

$$v_1 = v\frac{R_1}{R_2 + R_3 + R_1}$$

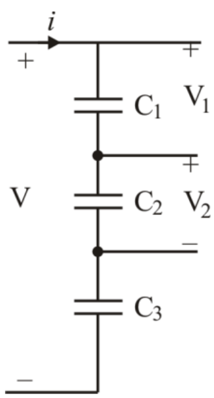
$$v_2 = v\frac{R_2}{R_1 + R_2 + R_3}$$

In voltage divider, voltage across one branch

$$= \text{Total voltage} \times \frac{\text{Resistance of that branch}}{\text{total resistance}}.$$

For Inductor: Same as resistor.

For Capacitor:

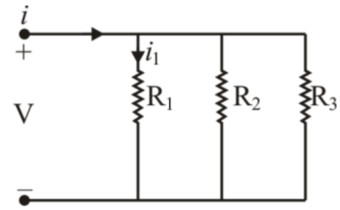


$$V_1 = V\frac{C_2C_3}{(C_1C_2 + C_2C_3 + C_3C_1)}$$

$$V_2 = V\frac{C_1C_3}{(C_1C_2 + C_2C_3 + C_3C_1)}$$

7. CURRENT DIVISION:

For resistor: A Parallel arrangement of resistors results in a current divider.



$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} \text{ and } i_1 = \frac{v}{R_1}$$

$$\frac{i_1}{i} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Thus $$i_1 = \frac{R_2R_3 i}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

For two branch current divider:

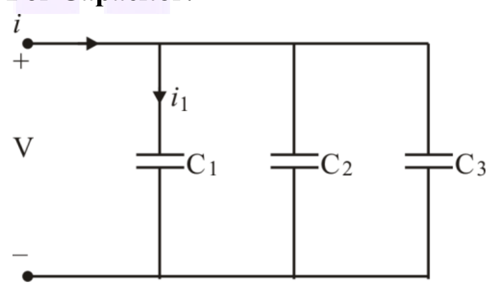
$$i_1 = \frac{R_2 i}{R_1 + R_2}$$

- For 2 branch circuit, the current in one branch is equal to the :

$$\text{Total current} \times \frac{\text{Resistance of Other Branch}}{\text{Total Resistance}}$$

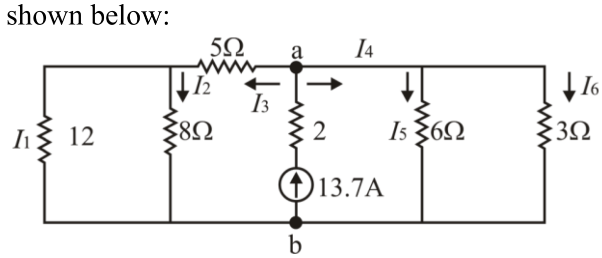
For Inductor: Same as resistor.

For Capacitor:

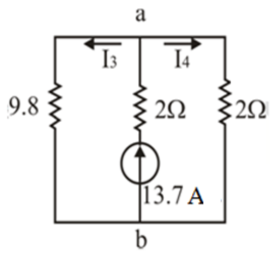


$$i_1 = i\frac{C_1}{C_2 + C_3 + C_1}$$

Example: Find all branch currents in the network shown below:



Solution: Circuit can be simplified as



$R_{eq.}$ (Left) $= 5 + \frac{12 \times 8}{20} = 9.8\Omega$

$R_{eq.}$ (Right) $= \frac{6 \times 3}{9} = 2\Omega$

Using current divider theorem:

$$I_3 = \frac{2}{9.8+2} \times 13.7 = 2.32A$$

$$I_4 = \frac{9.8 \times 13.7}{9.8+2} = 11.38A.$$

$$I_3 = I_1 + I_2$$

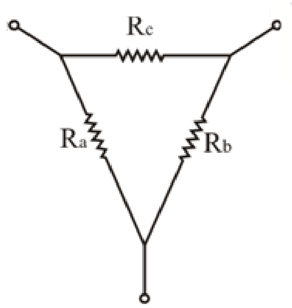
$$I_1 = \frac{8 \times 2.32}{12+8} = 0.93A,\ I_2 = 2.32 - 0.93 = 1.39A.$$

$$I_4 = I_5 + I_6$$

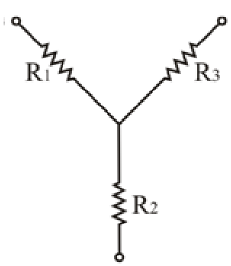
$$I_5 = \frac{3 \times 11.38}{3+6} = 3.79A. \quad I_6 = 11.38 - 3.79 = 7.59A$$

8. STAR-DELTA TRANSFORMATION (Δ↔Y)

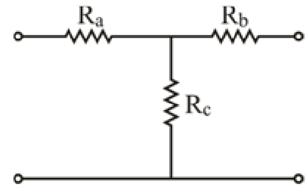
For Resistor:



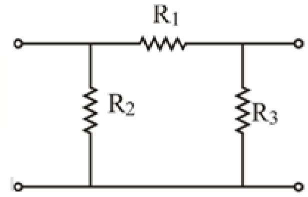
(Delta) Δ Connection



(Star) Y Connection



T Network



π Network

(a) **Delta to star Transformation:**

Also (π to T transformation)

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_b R_c}{R_a + R_b + R_c}$$

(b) **Star to Delta Transformation:**

Also (T to π transformation)

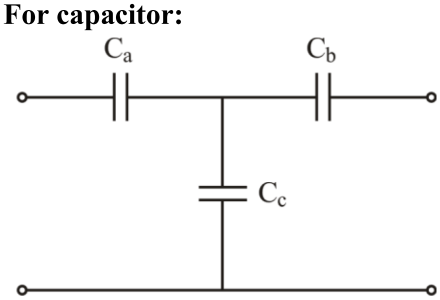
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

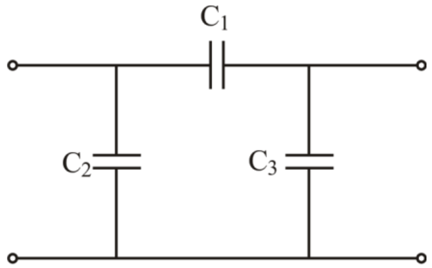
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

For inductor: Same as resistor

For capacitor:



T Network



π Network

(a) **Delta to star Transformation:**

Also (π to T transformation)

$$1/C_1 = \frac{\frac{1}{C_a}\frac{1}{C_b}}{\frac{1}{C_a}+\frac{1}{C_b}+\frac{1}{C_c}}$$

$$\frac{1}{C_2} = \frac{\frac{1}{C_a}\frac{1}{C_c}}{\frac{1}{C_a}+\frac{1}{C_b}+\frac{1}{C_c}}$$

$$\frac{1}{C_3} = \frac{\frac{1}{C_b}\frac{1}{C_c}}{\frac{1}{C_a}+\frac{1}{C_b}+\frac{1}{C_c}}$$

(b) **Star to Delta Transformation:**

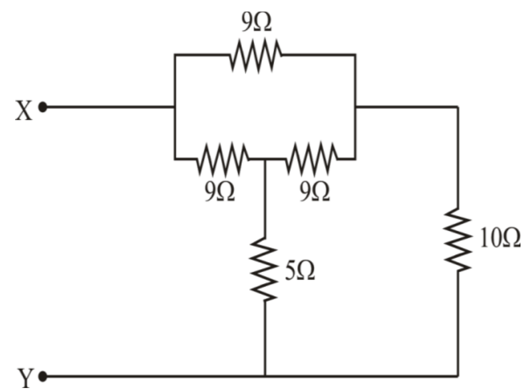
Also (T to π transformation)

$$\frac{1}{C_a} = \frac{\frac{1}{C_1}\frac{1}{C_2}+\frac{1}{C_2}\frac{1}{C_3}+\frac{1}{C_3}\frac{1}{C_1}}{\frac{1}{C_3}}$$

$$\frac{1}{C_b} = \frac{\frac{1}{C_1}\frac{1}{C_2}+\frac{1}{C_2}\frac{1}{C_3}+\frac{1}{C_3}\frac{1}{C_1}}{\frac{1}{C_2}}$$

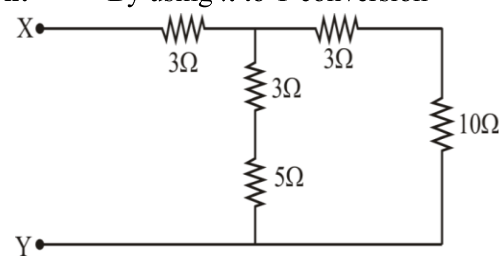
$$\frac{1}{C_c} = \frac{\frac{1}{C_1}\frac{1}{C_2}+\frac{1}{C_2}\frac{1}{C_3}+\frac{1}{C_3}\frac{1}{C_1}}{\frac{1}{C_1}}$$

Example:



Determine R_{XY}:-

Solution: By using π to T conversion

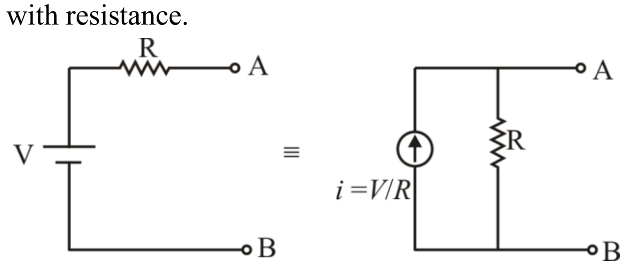


$R_{XY} = [(10+3) \parallel 8 + 3]$

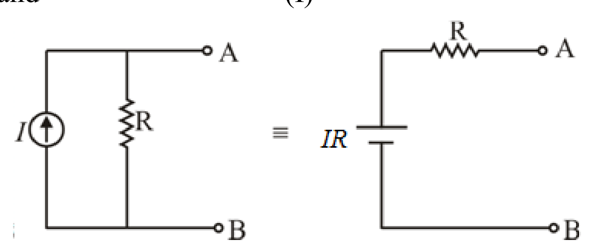
$R_{XY} = 7.95\Omega$

9. SOURCE TRANSFORMATION:

Voltage source in series with resistance can be converted to equivalent current source in parallel with resistance.



and (I)



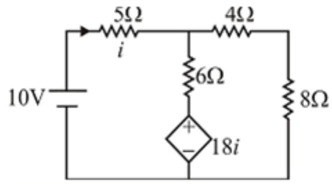
(II)

(I) Transformation of voltage source to current source.

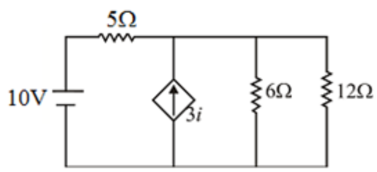
(II) Transformation of current source to voltage source.

Note: For dependent sources, source Transformation can also be applied. However, the dependent variables should be kept intact, since the operation of the dependent source depends on it.

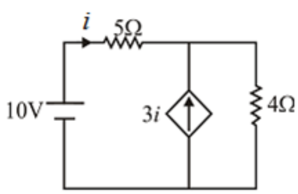
Example-: Do the source transformation of the following circuit:



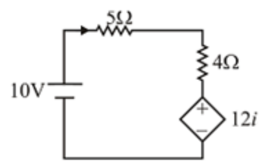
Solution:



Step-I



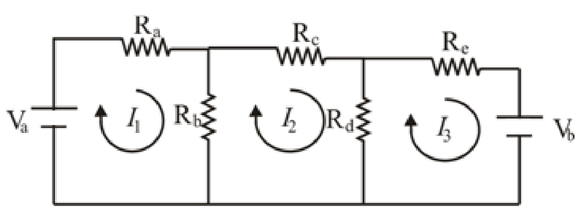
Step-II



Step-III

10. Mesh Analysis:

The mesh or loop method of analysis is explained by the circuit below:



Applying the KVL in 3 loops:

$$(R_a + R_b)I_1 - R_b I_2 = V_a$$

$$-R_b I_1 + (R_b + R_c + R_d)I_2 - R_d I_3 = 0$$

$$-R_d I_2 + (R_d + R_e)I_3 = -V_b$$

Writing then equation in matrix form:

$$\begin{bmatrix} R_a + R_b & -R_b & 0 \\ -R_b & R_b + R_c + R_d & -R_d \\ 0 & -R_d & R_d + R_e \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ -V_b \end{bmatrix}$$

Generalized mesh equation can be written as:

Impedance matrix $[Z][I]=[V]$ voltage matrix

$[I]=[Z]^{-1}[V]$

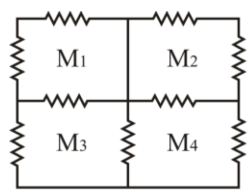
Thus current matrix can be obtained by taking the inverse matrix of impedance matrix and then multiplied by voltage matrix.

Key Point:

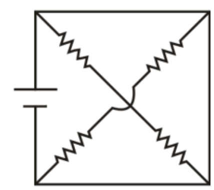
- The order of impedance matrix will be according to the number of meshes.
- Mesh is a loop and it does not contain any inner loop.
- Mesh analysis can be applied to only planar network.

Non Planar & Planar Network:

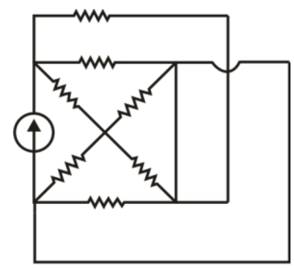
When the network is drawn on plane without any cross over, then the Network is called planar network.



Planar Network



Planar N/w

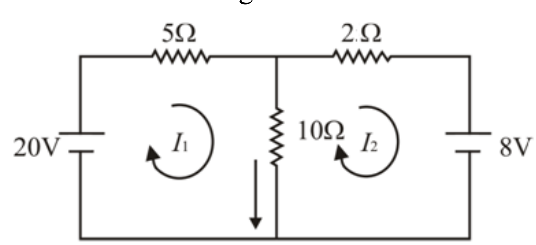


Non Planar Network

Procedure for Mesh analysis:

- Identify total no of meshes in the given network.
- Assign current direction for each mesh.
- Developed KVL equation for each mesh.
- By solving KVL equation find loop currents.

Example: Obtain the current in each branch of the network shown in figure:



Solution: Applying KVL in both meshes:

$-20+5I_1+10(I_1-I_2)=0$ .....(1)

$8+10(I_2-I_1)+2I_2=0$ .....(2)

On solving these equations:

$I_1=2A$

$I_2=1A$

Thus current in centre branch is $I_1-I_2=1A$

Note:

a. Total no. of equation required ⇒

$e=b-(n-1)$

b = no. of branches

n = total no. of nodes in the n/w

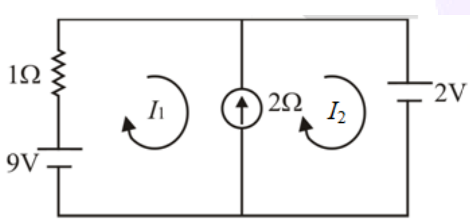
e = No of equation

b.

Total no. of *voltage* equation ,KVL=Total no. of Mesh

c. If two meshes are having a common current source (independent or dependent) then super mesh is formed. In super mesh techniques, those two meshes which has common source is treated as one.

Example: I_2



Super Mesh equation is:

$9-I_1-2=0$ (Do not write two separate Mesh equations)

Or $I_1=7A$

& $I_1-I_2=-2A$

Or $I_2=9A.$

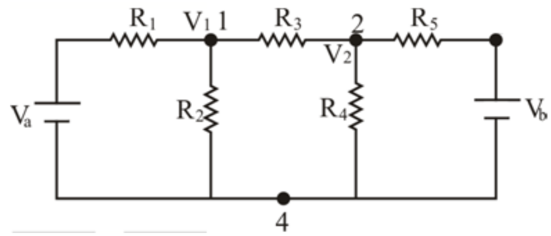
11. Nodal Analysis:

This analysis is applicable to both planar and non planar network.

In this method, one of the principal node is selected as the references and equation based on KCL are written at the other principal nodes.

Procedures of Nodal analysis

- Identify total no. of nodes in the given circuit.
- Assign a voltage at each node; one node is taken as reference node whose potential is assigned as zero.
- Develop KCL equation at each non reference node.
- By solving KCL equation find node voltage.



Applying KCL at each non reference node:

$$\frac{V_1-V_a}{R_1}+\frac{V_1}{R_2}+\frac{V_1-V_2}{R_3}=0 \quad \text{.......(1)}$$

And $$\frac{V_2-V_1}{R_3}+\frac{V_2}{R_4}+\frac{V_2-V_b}{R_5}=0 \quad \text{....(2)}$$

Putting in matrix form:

$$\begin{bmatrix}\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3}\\ -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}\frac{V_a}{R_1}\\ \frac{V_b}{R_5}\end{bmatrix}$$

Key Points:

- **SUPER NODE:-** A super node is formed by enclosing a dependant or independent voltage source connected between two node and any elements connected in parallel with it.
- **SUPERMESH**:- A super mesh results when there is a current source between two meshes.
- (No. of current equation = $N-1$), N = No. of nodes
- Nodal analysis → KCL + Ohm's law
- If the two non reference nodes have a common voltage source then super node technique is used.